Making math add up for students receiving special education

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To cite this article:

Ballin, A., Davidson, E., Caron, J., & Drago, M. (2022). Making math add up for students receiving special education. *International Journal of Whole Schooling*, *18*(1), 1-28.

Abstract

This article draws attention to the discrepancy between how math is taught in special education and general education classrooms, the consequences of this discrepancy, and how teachers can teach higher order math concepts to a wide range of learners, including students with moderate special needs, using empowerment math principles. Students assigned to special education are often taught a different kind of math from general education students, preventing them from practicing critical thinking skills and developing higher order thinking related to math concepts. The authors developed empowerment math based on years of teaching math to a diverse group of learners. The Nine plus One principles of empowerment math guide teachers to practices that promote inclusion and accessibility for a wide range of learners. The authors contend that too many students with moderate special education needs are being denied access to the STEM field, notably many students of color, when they are denied access to advanced level math education. Empowerment math could be used to remedy this discrepancy.

Keywords**:** learning disabilities, educational equity, special education, math education

Introduction

To learn how to add fractions, three students were pulled out of their general education class to receive “extra” instruction from a student teacher. In the general education classroom, the teacher used the curriculum “Math in Focus” to teach adding fractions. Students were taught the “why” of adding fractions, not just the “how.” This approach focused on critical thinking and problem-solving skills. In the special education classroom, the approach was different. The supervising teacher explained to the student teacher that pupils in the moderate special education classroom focus on “skills” such as adding and subtracting fractions, rather than the content from the general education curriculum.

The student teacher proceeded to give each student a sheet of paper that outlined the six steps for adding fractions. The fourth-grade students were expected to memorize the steps. They were instructed to make the “bottom” of each fraction the “same.” They were told that with the “new” denominator (rather than common denominator), they could now add the fractions. Non-mathematical language, such as “whatever we do to the bottom we have to do to the top,” dominated this lesson. The mathematical special education instruction methodology was rote memorization, practicing the same steps over and over until the student had them memorized. Understanding how to solve the problem was not required as in the general education classroom. Working hands-on with visual representations can support students in understanding what they are doing when adding fractions, and in becoming comfortable with the process, yet in this setting no manipulatives or diagrams were used to enhance comprehension. In this scenario, rote memorization differentiated special education math from general education math.

Why are students who are struggling with math being pulled out of the general education classroom and given a math teaching methodology that requires memorization and not conceptualization? The student teacher was merely delivering what the supervising teacher had instructed him to do as an effective way to teach these students with moderate special education needs. Why were these students being denied higher level thinking? Unfortunately, the trend of students with moderate special education needs receiving lower quality instruction based on teaching rote math has been documented (Blank & Smithson, 2014; Tan & Kastberg, 2017).

The goal of this article is to show teachers that they do not need a separate educational strategy to teach students with moderate special education needs; instead, they can hone their math strategies to include these students in the general education classroom. Additionally, this article aims to show the consequences of denying students with learning disabilities a quality math education, and to demonstrate how math that highlights critical thinking, problem solving, and analytical skills can and should be taught to students with disabilities. The authors use the term "math empowerment" to describe a pedagogical method of teaching math that promotes critical thinking skills and teaches for understanding. This math is designed both for students in general education and for students in moderate special education classes, so that all students can learn in one classroom together. There is no need to avoid high quality math instruction for students receiving special education just because students have been identified with a disability. In fact, there may be more reason to provide this kind of education. Gutiérrez (2013) described a sociopolitical "turn" to create a new mathematics education for all students. He emphasized the need for this shift, noting that the teaching of mathematics sits within a political and societal context that naturally involves power and domination privileging some over others. He explained, "Those who have taken the sociopolitical turn seek not just to better understand mathematics education in all of its social forms but to transform mathematics education in ways that privilege more socially just practices" (p. 40).

The authors of this article suggest that we need to empower students with mathematical education so that every child has access to high-level mathematical thinking skills. Math empowerment means that all students are taught math for understanding. Gutiérrez (2013) noted that we should judge math students on creative problem solving rather than on getting the right answer. Salman Kahn, founder of Kahn Academy, suggested that we teach for mastery (2016). He noted that typically if students do not get the right answers on a test, the teacher often just moves on. The student might get a "B" or a "C," but the material is never mastered. As the student continues with math, there are gaps, and over time the student gets tracked to lower classes and develops the identity of a poor math student. With empowerment math, all students are given the opportunity to master the content because the teacher takes responsibility for making sure that all students understand the higher-level concepts, so that the students can use these concepts as they progress in their math education. When we teach to all students, we no longer need to assign students to lower math classes, relegate them to a pull-out special education math class, or give students the identity of a "poor math student" while others are given the identity of a "talented math student."

Literature Review

In 2000, the National Council of Teachers of Mathematics (NCTM) outlined instructional strategies for teachers of mathematics. Another set of standards, the Common Core, was adapted in 2015 under the authority of the Council of Chief State School Officers and the National Governors Association (Common Core State Standards Initiative, 2022). In both of these sets of standards, students of mathematics are required to use higher level thinking to draw inferences, interpret numbers, and explain their reasoning.

Unfortunately, instructional time in a moderate special education classroom does not always include higher level mathematical analysis. In a study of 50 schools and over 300 cross-curricular teachers, Blank and Smithson (2014) found that instruction for students with disabilities included less time on “analysis of information, and evaluating evidence and arguments, and more time on test preparation” (p. 143). This study suggested that students in special education classes were not receiving a proper standards-based education that focuses on critical thinking skills. In their mathematics classrooms, these students spent more time following procedural math and preparing for tests than their peers (Blank & Smithson, 2014). Thus, students with disabilities are denied the opportunity to learn specific mathematical skills and concepts at the same depth compared to students without disabilities, creating inherent educational inequity (Tan & Kastberg, 2017). Gottfried et al. (2016) noted that the differences in math scores between students labeled with a disability and those without can be observed as early as fourth grade, and that this gap only grows larger as they progress. Gottfried et al. posited that secondary special education teachers acknowledge a weakness in teaching mathematics, and that students with disabilities perform better when they are in inclusive classrooms. Additionally, Wei et al. (2017) claimed that students with Autism Spectrum Disorder (ASD) are more likely to major in fields related to Science, Technology, Engineering, and Mathematics (STEM) if they are in inclusive classrooms.

The consequences for the special education population, and for the nation, are greater than they may appear, because there could be many more students contributing to science and math fields if all students had a comprehensive math education that includes analytical skills and problem solving. Further, research shows that the population of students with disabilities is disproportionately dominated by students of color (Harry & Klingner, 2014; Hutchison, 2018;

Oakes et al., 2018). Therefore, students of color may be disproportionately denied access to higher level mathematical courses. Students of color receiving special education services are also more likely to be removed from the general education classroom compared to their white counterparts with the same diagnosis, and to receive a lower quality educational program (Annamma et al., 2016). School tracking, the process of placing students into different classrooms based on perceived ability, is subject to bias and discrimination and therefore places more students of color in lower-level math and science courses (Leonardo & Grubb, 2019; Oakes et al., 2018).

For students of color, entrance into special education can have grave consequences for educational success, especially in terms of access to mathematics and the entire STEM field. Both students of color and students with disabilities are underrepresented in the STEM field (Gottfried et al., 2016; Wei et al., 2017; Zimmer et al., 2018). These factors suggest that students of color with disabilities may be at a larger disadvantage to enter the STEM field than their white peers.

Reversing the inequities experienced by students receiving special education and students of color can require radical measures. For example, Dr. Adrian B. Mims, Sr., created The Calculus Project to address math inequalities at the high school in Brookline, Massachusetts, a high-performing district with a mathematical achievement gap between white students' and Black and Brown students' scores (The Calculus Project, 2022). To address this achievement gap, The Calculus Project developed a five-year "coordinated, research-supported" plan that included six components: pre-teaching in the summer and tutoring during the year; including a "critical mass" of students of color and from low-income backgrounds in high-level math classes; after-school study groups; curriculum explicitly teaching students about STEM professionals of color; paid "'peer teaching' opportunities"; and transitional and college academic support. Ultimately, almost every student in Cohort I successfully completed Honors and AP calculus, and now these students attend highly ranked US colleges, with subsequent cohorts "demonstrating similar success" (The Calculus Project, 2022).

Since mathematics is considered a gateway to higher level STEM courses in high school, it is not surprising that students with disabilities are underrepresented in STEM fields (Gottfried et al., 2016; Losinski et al., 2019). Spooner et al. (2017) highlighted the shift in mathematical instruction for students with disabilities, explaining that "teaching mathematical problem solving to students with severe disabilities is a relatively new topic. It is of utmost importance that we as educators maintain high expectations and do not place a ceiling on the mathematical learning potential of students with severe disabilities" (p. 184). From the noted lack of U.S. science education documented when the Russians launched Sputnik ahead of the US (Sleeter, 1986) to today's economic climate, concerns that students in the United States are falling behind in STEM fields are widespread, and a specific student population with potential contributions is being denied the needed education to participate (Gottfried et al., 2016; Wei et al., 2017; Zimmer et al., 2018).

Problem solving, critical thinking and inquiry skills are all typically taught in higher level math courses. These skills are necessary in most professions and are often denied to students with disabilities if they are not taking higher level STEM classes where these skills are incorporated into the class (Losinski et al., 2019; Zimmer et al., 2018). Even further, students in special education are considered a risk factor for entering the "school-to-prison pipeline" (Mallett, 2017, p. 563). Improving math outcomes for students receiving special education services and students of color can have a positive impact on these populations.

Empowerment Math

What Is Empowerment Math?

Teaching empowerment math means teaching for understanding and giving students access to higher level thinking, such as making inferences and interpretations. When teachers teach math in a routine of completing examples in class, asking students to answer problems similar to the example, reviewing the answers, and moving on when the topic is "covered," they are missing an opportunity to develop higher level thinking skills. Teaching with empowerment math means engaging students using strategies such as the Nine + One Principles. The authors have established these principles to create a math framework that allows both students and educators to feel empowered in their math thinking, and to gain deeper understanding. The Nine + One Principles are not meant to be an exhaustive list. While other approaches to math education exist that teachers may find equally fundamental, these are the principles the authors have found through practice to be most important.

The empowerment math way of teaching is not easy. It may take students longer to grasp material and require more time for the teacher to prepare, analyzing student errors and targeting conceptual understanding. Finally, it may involve a new way of thinking that requires students to master the material before entering a new topic, something that is not standard in every classroom. This might mean that students in one class are not working on the same concepts at the same time, yet teaching in this manner allows students in special education the same opportunities as students in general education, in terms of access to the material and higher thinking skills.

Nine + One Principles of Empowerment Math

1. ***Flexibility: Find multiple ways of teaching one concept.*** If a student is having trouble learning a particular algorithm or procedure, teachers can select a different algorithm that better matches the learner. A successful algorithm needs to be accurate, efficient, and flexible (Russell, 1999). For example, multi-digit addition, including computation, traditionally involves regrouping or trading; however, it does not need to be taught from right to left. Consider the problem 27 + 48 written vertically as students might see it in second or third grade. Traditionally, students are usually taught to add the ones column and then “carry” into the tens and then add that column. An alternative method has students first add the tens, then add the ones, and lastly add those together. This method keeps the place values intact and works from left to right. First, add 20 + 40 = 60. Then add the ones: 7 + 8 = 15. Add those answers together and the total equals 75. Students write left to right in English, and they read numbers left to right, so having the computation work directionally in the same way as reading and writing numbers can make more sense for students with dyslexia. Additionally, for students who struggle with place value, this method reinforces place value rather than ignoring it, giving extra practice with a difficult and important mathematical idea.

$$\begin{array}{r} 27 \\ +\,48 \\ \hline 60 \\ +\,15 \\ \hline 75 \end{array}$$

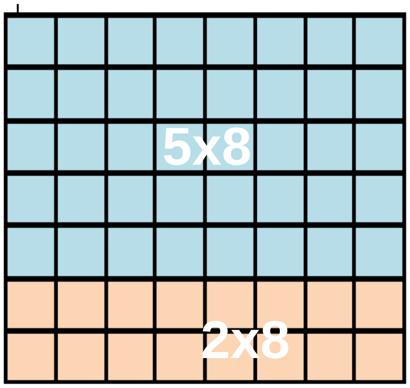
2. ***Foundational knowledge: Use conceptual mathematics instruction in the early years to build a foundation.*** In elementary math, children often think the “equals sign” means “the answer is coming up,” rather than that the value of everything on one side of the equal sign is worth the same as the value on the other side. Understanding this concept is essential to higher math. Starting in grade one, teachers can introduce equals in multiple ways, all based on this concept of equality. For example, 7 = 7 and 2 + 5 = 7 and 7 = 6 + 1 and 3 + 4 = 5 + 2 are all statements of equality. Young children can try out these equalities with blocks and

scales, building each and noting how they balance. This will serve them well when they begin algebra, an essential mathematical concept (Usiskin, 2005). For students who may struggle with memory issues, they need as much reinforcement and practice as they can get. By starting out with the correct understanding of the equal sign, the concept of equality is being reinforced algebraically from the beginning. Students are not being taught incorrectly but are consistently getting the correct exposure to what the equal sign means.

3. ***Modalities: Remain open to multiple modalities matched to learners.*** Students need entry points matched to how they learn. Solving problems kinesthetically makes mathematical thinking *make sense* for learners who need to move around (Street & Baker, 2006). For example, some learners can more easily solve word problems by acting them out. Many learners can better access word problems if they use actual objects rather than cubes or blocks to represent those objects, e.g., pencils for word problems about pencils. Students receiving special education are often more concrete, so using real objects for what they are is easier than working with representations. Diagrams, charts, and tables can allow more visually oriented learners to access curriculum more effectively than through written words.
4. ***Sense-making: Develop a mindset of connections.*** Many learners, especially those who have been taught by rote instruction, come to believe that “math is magic.” When some of them ask questions about why math works, they are told, “Don’t worry about it, just do it, and it gets you the right answer.” Instead, teachers could respond, “Good question, let’s figure out what’s going on here.” Together, teacher and students could then analyze the process involved, exploring connections to related concepts. When students develop a mindset of sense-making, they are building a more solid foundation for learning higher level

mathematics. Additionally, some students with disabilities who are concrete thinkers need to engage more in sense-making to move beyond overly concrete or rigid thinking.

5. ***Integration: Integrate concepts and procedures so they complement one another.*** Integrating knowledge of multiplication facts with an understanding of basic principles of multiplication can be useful, helping students develop skill with efficient fact recall while exposing them to the *distributive property*. The underlying idea that multiplication involves multiple groups of the same size is essential knowledge for higher mathematics. In this visual example, breaking up a difficult math problem into multiple easier problems can increase math fluency and make multiplication easier. Students can turn the "hard to remember fact" of 7 x 8 into two of the "easiest facts to remember" by adding 5 x 8 and 2 x 8, which are easy facts, to get to 7 x 8. Additionally, drawing out a visual representation can help with comprehension. Students who struggle with mathematics often get discouraged when trying to memorize multiplication facts, but can access multiplication more easily when integrating the distributive property. For students who struggle with memorization, this method gives them the opportunity not to have to memorize as many facts. This can be very helpful and less stressful.



6. ***Deliberateness: Choose numbers deliberately to allow easy access.*** Sometimes when teachers are creating or choosing worksheets, they are diligent about the language they use for word problems, but less so about selecting numbers for exercises. For example, when adding fractions, placing the focus on the fundamental idea of common denominators provides an excellent opportunity to choose deliberate examples. The introduction can begin

with fractions that already have common denominators, e.g., 1/8 + 3/8. Once this pattern is mastered, a next step is adding fractions where one of the two denominators provides a common denominator, e.g., 1/2 + 5/6. For the next examples, fractions can be selected where the common denominator will be the product of the two denominators, e.g., 3/4 + 2/5. After gaining facility with these scenarios, students can go on to find common denominators for more challenging problems like 3/8 +5/6. Students receiving special education services tend to get more discouraged because they may struggle more academically. Choosing numbers carefully allows students to be successful in the process sooner and more often.

7. ***Scaffolding for cognition: Scaffold both to give access and to build independence.*** It can be tempting to "support" students who struggle by *leading* them to the answer. Often this is done subtly so students feel successful but, in truth, have not had the opportunity to do the thinking. This type of teaching can be gratifying in the moment but deprives students of real learning. Instead, scaffolding should give students access to problems and allow them the opportunity to do the *thinking* necessary to solve problems. Scaffolding breaks problems into smaller steps, which can make them less overwhelming. Often students with special needs suffer from anxiety and can be easily frustrated. This kind of scaffolding can ease both anxiety and frustration (Kusmaryono et al., 2020).

8. ***Diagnostic teaching: Affirm student strengths and address misconceptions.*** When a teacher understands patterns in mistakes a student is making, the teacher can address the student's misconceptions specifically, helping the student gain new conceptual understanding as well as procedural competence. For example, if a fifth-grade student is getting about 80% right on multi-digit addition and subtraction worksheets, a vigilant teacher might uncover that the child's errors occur specifically when the minuend has a zero

or several zeroes, whereas the student could regroup successfully in all other circumstances. In this example, the teacher can help the student regroup using a strategy she is familiar with by applying her understanding to problems with zero in the minuend, targeting one specific misunderstanding and building on student strength. Addressing this targeted misconception means that the child becomes successful in subtraction and acquires a deeper conceptual understanding along with more accurate procedural skills. Sometimes for students in special education, if they are getting “most” problems correct, both students and teacher can be relieved. However, they could be missing a simple misconception that can impact learning as the child moves on in math.

9. ***Vocabulary instruction: Explicitly teach math language with visuals.***

Teaching accurate mathematical terms increases consistency, so that students are hearing repeated mathematical language. Internalization will accompany students as they progress in math. For example, when teachers use a term like “equation” rather than “number sentence,” students can recognize the word “equals” as part of its structure, emphasizing the important idea that an equation shows the quantity on one side is equal to the other. The term “number sentence” does not contain this morphological cue. To enhance internalization of vocabulary, teachers can use visual cues and color to make words easier to remember, post words for the current unit in front of the room, and keep previous unit vocabulary words on a mathematics word wall, as shown in the following examples.

+1. ***Teaching teachers: Support this challenging work.*** Empowerment math can be challenging to learn and to implement. Mathematics educators need to provide pre-service and in-service teacher training. While it is the responsibility of teachers to develop this version of instruction, it is *not the teacher's fault* if they do not yet have the resources and skills. Ellen, who is a math specialist and has instructed teacher candidates for 31 years, vividly remembers when she provided professional development for teachers at a school for students with emotional and behavioral challenges. The teachers were using an easily accessible mathematics program that relied on a kinesthetic approach not connected with mathematical meaning. Children were having some success and feeling good, but were completely unprepared for the high-stakes tests that involved a deeper kind of thinking than the instruction they had received. They could not answer the questions and were frustrated. Teachers need a curriculum that is going to match the kind of math that students need to learn. This kind of math can be harder to teach, so educators need support as well.

Why Is Empowerment Math Instruction Important?

As teachers, we have a major responsibility for helping to prepare our students to be active and thoughtful citizens of a democracy. To be this kind of citizen, a student must be able to think critically and logically about the many issues confronting a democracy. Citizenship involves understanding what we hear and see. It involves being able to follow arguments, with and without numbers, and being able to challenge those arguments. In the domain of mathematics, being a thoughtful citizen also involves understanding data and statistics to avoid being misled by people who take advantage of those who have weak mathematical thinking.

Beyond being a thoughtful citizen, all of our students need to learn mathematics to help them with practical, real-world skills like budgeting, sound fiscal management, and problem solving. All of our students need to learn mathematics to help them with potential skills in the world of work, including measuring, estimating, and handling money, for jobs ranging from construction and carpentry to marketing, finance, and other career areas. Many of our students need math instruction that will serve as a foundation for learning higher mathematics. This involves having a conceptual understanding of mathematics rather than just rote memorization of computational algorithms. Being successful in math also involves developing cognitive flexibility, the ability to learn in different contexts and in different ways. Simplifying mathematics--especially for students receiving moderate special education services, and often in the name of kindness--to rote procedures so that learners "feel a sense of accomplishment" and even perhaps a feeling of mastery, may lead to a conventional definition of "success," yet it is not success that, in the long term, serves our students.

Empowerment Math Examples in Elementary, Middle, and High School Classrooms

In this section, we provide selected examples of how empowerment math can be taught at various age levels, highlighting specific Nine + One principles.

Empowerment Mathematics Instruction in an Elementary Classroom

Using empowerment math, we want teachers to reach a range of learners. Teachers can combine Principle #4, Sense-Making, and Principle #8, Diagnostic Teaching, by conducting clinical interviews with individual children so they can build on their strengths and understandings and address their misconceptions. Educators can then plan instruction with several guidelines to increase sense-making. What is at the essence of the mathematics itself? What algorithms reinforce this mathematics and are easier for students to learn? What properties? For example, what happens when we multiply by zero or one? Children should *not* memorize these facts. Instead, we need to be sure they understand that one is the identity element in multiplication. Multiplying by one does not change the value of a number. Multiplying by zero always gives zeros, so there is nothing to memorize but a concept to understand.

Deliberateness, Principle #6, is easily demonstrated when looking at teaching multiplication facts. Instead of learning multiplication facts “in order” from zeros to ones, twos, and onward to tens, these facts are more accessible if students learn them from “easiest to hardest.” Tens are easy because when we multiply by ten, we “bump a number over to the next place value.” After tens, fives are easy and then twos. The nines can be accessible to most learners using a variety of strategies, some more kinetic and others more conceptual.

As teachers guide students along this deliberate path, taking advantage of commutativity, students end up with six facts that can be more challenging: 6 x 6, 6 x 7, 6 x 8, 7 x 7, 7 x 8, and 8 x 8. This is the time to introduce the distributive property, which demonstrates Principle #5, the integration of concepts and procedures. While 7 x 8 can be one of the hardest facts, using the distributive property to create two groupings, (5 x 8) + (2 x 8), presents students with two easy facts that are then simpler to add together. This process also provides students with conceptual practice in taking numbers apart and putting them back together again. Once students who typically struggle in mathematics develop strategies for quickly accessing facts, they are more ready to dig deeper into understanding the essence of the operations, in this case multiplication, building a stronger foundation as noted in Principle #2, Foundational Knowledge. With this empowerment they are then more ready to learn algorithms well matched to themselves as learners.

There are several multiplication algorithms better suited to some learners than traditional methods, because they consistently reinforce place value. These alternatives also allow students

```
   5 4
   4 3
   587
  x 65
  2935
+35220
38,155
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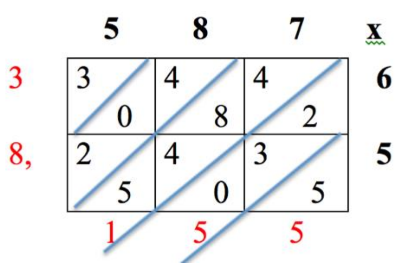
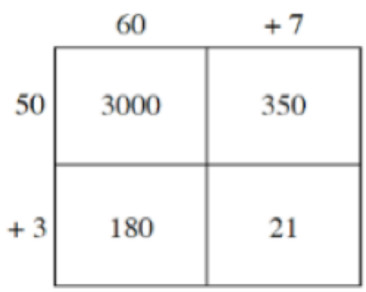
to do all their multiplying and then all their adding, rather than constantly moving back and forth between multiplying and adding. Principle #1, Flexibility, explains the rationale for offering these alternatives. In traditional US multiplication instruction, multi-digit multiplication looks like the example given here. First we multiply 5 x 7. Then, we usually add the carried number to the next numbers multiplied and continue in this fashion, ending with adding 2,935 and 35,220.

Many current curricula teach multi-digit multiplication using procedures that make more sense to most learners, including those with special needs. One version is called *lattice*



multiplication. In lattice multiplication, students first fill in all the pairs of multiplication facts in their correct boxes based on place value, as shown in the example to the left, which multiplies 587 x 65. Then students add each place value on the diagonal, again working from right to left. The answer appears on the outside of the box, read vertically, then horizontally, from left to right. The steps are easier with the visual organization because students are doing one operation at a time, just multiplying and then just adding, not switching back and forth between operations.

Another effective procedure for teaching multi-digit multiplication in inclusive classrooms is *generic rectangles* or the *box* method. This method reinforces place value and provides a visual organizer. It also uses the distributive property by breaking apart 67 and 53 into tens and ones, providing foundation for essential algebra skills. As with lattice, first students do all the multiplying, then add the four partial products together to get 3,551.

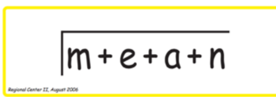


In another embodiment of the principles of empowerment math, students can access both concepts and skills kinesthetically through learning about *measures of central tendency* (averages). First, teachers can use data that captivates students to encourage sense-making, Principle #4. Take the example of how many letters are in student first names. What do students predict will be the most common number of letters? After discussing their predictions, students group themselves by the number of letters in their names. Which group has the most? This is the *mode* or *most common*. Often this data set will be *bimodal*, with a tie between two numbers.

Next, students can line up from the shortest name to the longest. Who is in the *middle*? How many letters long is their name? This is the median. One way to determine the *middle* is to have one student on each end leave the line at the same time, then the next student on each end leave, and so forth until one is left in the center. If there is a tie, the median is the number halfway between those two.

The third measure is the *mean* or traditional average. Continuing with using student names, each student takes the number of blocks for the letters in their name. Students stand in a circle and, one by one, each student tosses his or her blocks into the bowl in the center. Then these blocks are distributed, one to each student. Then, a second block is handed to each student and so forth. If this works out exactly, then each student now has the *average* number of blocks. If there are leftover blocks, teachers can either move into a discussion of decimals or fractions to complete the mean, round off, or describe the mean as “between 5 and 6,” depending on the age or level of the students. This set of activities also lends itself to producing bar graphs and pie graphs, on paper or perhaps hands-on, using the children themselves to demonstrate relationships of height or other visual data. Teachers can also incorporate data that is part of social studies and science units, thus helping students better master that content as well as the mathematics.

As explained in Principle #9, Vocabulary Instruction, sometimes children understand the content of a topic but become confused with the vocabulary. This is common with measures of central tendency, a case where visuals such as the following examples can be helpful.



Principle #9 emphasizes the importance of understanding math vocabulary, often a struggle for students who have trouble learning mathematics. Having a consistent and appealing strategy to teach mathematical vocabulary allows students to engage in the content of the mathematics. Using this strategy, and others among the Nine + One Principles, all elementary learners, including students in special education classes, can develop a robust conceptual foundation for mathematics that will strengthen their future learning.

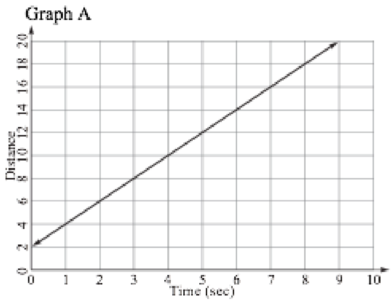
Empowerment Math Instruction in the Middle School Classroom

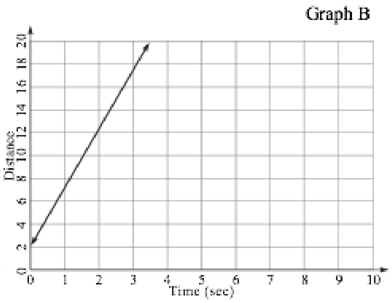
Teachers can also empower middle school students (students in both general education and special education together in an inclusive classroom) to connect to mathematical material by using the Nine + One Principles. In this section, examples of lessons on understanding how functions have different rates of change demonstrate a number of the principles.

A useful resource for Principle #4, Sense-Making, is provided in math educator Andrew Stadel's website *Estimation 180*. This website provides visual daily estimation challenges that use real-life examples, where students support their thinking with evidence (Stadel, 2022). For example, students can look at a picture of a basketball court and conceptualize the distance walking across the court to make sense of a y-axis. Asking "How long would it take to walk across the basketball court?" can help students think critically about the line of movement. Because students can relate to this example through their own experiences, they tend to gain easier access to the concept of ratio and rates between time and distance. Using Principle #7, Scaffolding for Cognition, teachers can build further on the subject. A teacher can ask students to write estimates on the board. Then, students are compelled to compare those estimates to rates of speed to see which estimate is closest.

Along with using concrete visuals such as the ones from Stadel's *Estimation 180* website, scaffolds like those posted on Dan Meyer's (2007) *Graphing Stories* math website can also enhance foundational knowledge, connect students with material, and further support complex conceptual mathematics. Giving students a graph template, the teacher can play a video of a man walking across the court (as in the photo shown here), and ask questions like, "When does the man cross half court?" Then, the teacher can replay the video at half speed and have students do a think-pair-share of their graphs. By playing the video multiple times and at different speeds, students have more opportunities for success. Every student can compare their graphs to others and learn from each other in terms of which details they need for their graph. These supports provide intensive and multimodal scaffolding, thus also enacting Principle #3, Modalities.

In another activity called "How Should I Move" (Chaikin, 2022), posted on NCTM's *Illuminations* math education website, a motion detector is used to create the graphs below. By





incorporating the students' movements into this activity, students use mathematical reasoning to adjust their movements to make these graphs. They can see how the graph is made and use reasoning skills to create different graphs, again encouraging Principle #4, Sense-Making.

As these different principles are used to help students understand rate of change, it is important to provide explicit teaching of vocabulary during the lesson, drawing on Principle #9. The teacher must connect the students' reasoning to words like "slope" and "rate of change." For students to internalize how to find slope, they should understand the related mathematical words. Displaying these terms on a word wall with visuals and color can serve as a reference as students learn.

Using the Nine + One Principles empowers both students in general education and special education to learn math in an engaging way so that everyone can be a problem solver. Just as these principles were used to build a robust foundation at the elementary level, this section shows how the same principles can be used to solve more complex problems. In doing so, students discover that they are all capable of higher level thinking.

Empowerment Math in the High School Classroom

An example of incorporating the Nine + One Principles at the high school level can be seen in the teaching of trigonometry, a highly challenging topic often assumed to be beyond the scope of students receiving special education. In empowerment math, we want students to use their reasoning skills when interpreting trigonometric functions, not their memorization skills. Because the unit circle is so important, traditionally students are expected to memorize its components. Since this topic requires so many skills, using Principle #8, Diagnostic Teaching, is important. What is the student's understanding of reference angles in a circle? What is their skill level with special right triangles? Have they mastered trigonometric ratios? Collecting data and then addressing learners' misunderstandings will be needed for success with this topic.

To approach this complex topic in a way that engages students and does not push them to memorize, an empowerment math class could construct a unit circle on a larger scale, implementing a kinesthetic-based mode of instruction using Principle #3, Modalities. In a school with a large floor area with tiles, the class could use the tiles as a grid. Outside, students could use sidewalk chalk on a large area of pavement, or a circle drawn on a soccer field. Physically moving around a large circle and constructing the key components encourages students to use reasoning skills, instead of copying notes and filling in a blank unit circle. In addition, the activity allows students to move and learn kinesthetically.

Constructing a large-scale unit circle offers opportunities to differentiate based on students' needs. Multiple strategies can be used to approach the unit circle, depending on each class's strengths and weaknesses, incorporating Principle #8, Diagnostic Teaching. Perhaps students need to practice special right triangles; if so, they will use the triangles for each quadrant. On a conceptual level, students are setting up the reference triangles and reasoning the distance along each axis. If students need practice with the circle's radians, they can make their way around the circle labeling the radians, counting around the circle by third radians and quarter radians. To label the coordinates on the unit circle, they can physically move along the x and y axes to determine each point based on what they know about special right triangles and radians.

This scaffolded, multi-modal approach (Principles #7 and #3) allows all students, including those from special education backgrounds, to access this rigorous math topic. The use of kinesthetics, and a teacher diagnostically approaching a student's understanding (Principle #8) help to empower students in their math education, ideally building on the foundational strengths (Principle #2) they have developed in inclusive classrooms at previous levels.

Conclusion

Math instruction for students identified as needing special education appears, in many cases, to follow a different path from students in general education math classes. Too many students are being denied a meaningful mathematical education because teachers have not learned how to teach to the wide range of students' learning needs. The Nine + One Principles of empowerment math can help to make math accessible for all learners. We acknowledge that teaching with these principles is not easy and takes time to master. Yet, we suggest that it is imperative for teachers to embrace the principles as one step toward educational justice. The example of Albert Einstein as learning disabled is often put forward to exemplify the capability of students with disabilities, but if Einstein had been born in this century, would he have been denied a rigorous mathematical education? How might this have changed what we know today? How many more Einsteins are we not seeing in the STEM field due to our current system of teaching special education math? Empowerment math can aid all students to pursue their talents no matter their label or diagnosis.

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